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Near-Wall Treatment for Turbulent Boundary-Layer Computations

R. Arora* and K. K. Kuo†
The Pennsylvania State University, University Park, Pennsylvania
and
M. K. Razdan‡
Exxon Research and Engineering Company, Linden, New Jersey

Abstract

A BOUNDARY-LAYER flow model utilizing the k-ϵ closure and a special near-wall treatment has been developed to predict flow profiles under various conditions. The modified van Driest formula suggested by Cebeci and Chang is used in the near-wall region. Comparisons have been made with several sets of experimental data, including planar or axisymmetric, incompressible or compressible, laminar or turbulent, with or without blowing, reacting or nonreacting, and subsonic or supersonic flows. In most cases, computed results agree closely with experimental data, indicating that the model with the specified near-wall treatment is suitable for a wide range of boundary-layer flows.

Contents

The k-ϵ turbulence models[1] (based on k, turbulent kinetic energy, and ϵ, turbulent dissipation rate) are increasingly used in finite-difference calculations for predicting turbulent boundary-layer flows. Treatment of the wall region deserves special attention because of the existence of local steep gradients and the fact that turbulence models for k and ϵ are not applicable in the low Reynolds number region. The objectives of the present study were 1) to develop a treatment for the near-wall region to be used with the k-ϵ equation model, 2) to test the predictive ability and range of application of the numerical code, and 3) to check the universality of the set of turbulence constants suggested in the literature.

In the present analysis, k and ϵ equations are applied to a region that begins a small distance above the wall ($y^+ = 15$) and continues to the edge of the boundary layer. Thus, boundary conditions for the two equations must be applied to the region away from the wall rather than at the wall in order to avoid the low turbulent Reynolds number region near the wall, where closure models for k and ϵ are not valid. At the edge of this region, however, it is reasonable to assume that the production and dissipation terms of the k equation are dominant and can therefore be equated.[2] This yields

$$\epsilon = \frac{\mu_t}{\bar{\rho}}\left(\frac{\partial \bar{u}}{\partial r}\right)^2 \tag{1}$$

Turbulent viscosity μ_t close to the wall is calculated using a modified van Driest's formula[3]

$$\mu_t = \bar{\rho}\,[\ell \mathfrak{D}(y_w + \Delta y_w)^2]\left|\frac{\partial \bar{u}}{\partial r}\right| \tag{2}$$

where $\ell = 0.41$ (von Kármán's constant), y_w is the distance from the wall, Δy_w is defined[4] as

$$\Delta y_w = 0.9(\mu/\bar{\rho}u_*)\,[\sqrt{R_h^+} - R_h^+ \exp(-R_h^+/6)] \tag{3}$$

and the dimensionless roughness R_h^+ is defined as $\bar{\rho}u_* R_h/\mu$.

The damping coefficient $\mathfrak{D}$ is altered to include the effect of surface roughness and local shear stress, as suggested by Cebeci and Chang[4] and Baker and Launder,[3] respectively:

$$\mathfrak{D} = 1 - \exp\left[\frac{(y_w + \Delta y_w)\bar{\rho}u_*}{A^+\mu}\,\frac{\tau}{\tau_w}\right] \tag{4}$$

μ_t is also related to k and ϵ, as suggested by many researchers,[1]

$$\mu_t = c_\mu(\bar{\rho}k^2/\epsilon) \tag{5}$$

By eliminating μ_t from Eqs. (1), (2), and (5), k and ϵ can be expressed as

$$k = \frac{[\ell \mathfrak{D}(y_w + \Delta y_w)]^2}{\sqrt{C_\mu}}\left(\frac{\partial \bar{u}}{\partial r}\right)^2 \tag{6}$$

$$\epsilon = [\ell \mathfrak{D}(y_w + \Delta y_w)]^2\left|\frac{\partial \bar{u}}{\partial r}\right|^3 \tag{7}$$

The near-wall treatment of the k and ϵ equations presented here is similar to that of Chambers and Wilcox.[5] The need for such a treatment is apparent in view of the reasons mentioned earlier. Although this method is not unique, it does have as its logical basis the assumption that production and dissipation terms are dominant at the edge of the near-wall region. The values of k and ϵ determined from Eqs. (6) and (7) are consistent with their distribution above the near-wall region.

A literature survey was made to collect reported experimental data relating to boundary-layer flows. Experimental conditions were simulated in the computation by specifying actual physical geometry and boundary conditions. Some of the results of our comparison are presented in Table 1 and Figs. 1-4. (See Ref. 2 for more detailed information.)

In conclusion, it appears that the k-ϵ model with near-wall treatment presented here is adequate for boundary-layer calculations. The set of turbulence constants used[2] was found to be applicable to a wide range of flow situations, including chemically reacting turbulent boundary-layer flows.

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*Research Assistant, Mechanical Engineering Department. Student Member AIAA.

†Professor, Mechanical Engineering Department. Associate Fellow AIAA.

‡Senior Research Engineer. Member AIAA.

Table 1 Summary of results of comparison with existing experimental data

Researcher	Type of turbulent boundary-layer flow	Results of comparison
Klebanoff[6] (Fig. 1)	On flat plate at zero incidence	Overall comparison is good; slight discrepancies, but within experimental error
Laderman[7,8] (Fig. 2)	On flat plate in supersonic flow with zero pressure gradient. Adiabatic or isothermal walls	Good agreement. Predicted profiles are independent of wall temperature, as experimentally observed
Sandborn[9] (Fig. 2)	"Best estimate" based on various investigations. Mach numbers ranging from zero to hypersonic values	Most of the predictions lie within Sandborn's best estimate
Laufer[10] (Figs. 3 and 4)	Fully developed turbulent pipe flow	Good agreement except for μ_t, which is slightly underpredicted at $y_w/R > 0.6$
Nunner[11] (Fig. 4)	Fully developed turbulent pipe flow	Close agreement in most of range

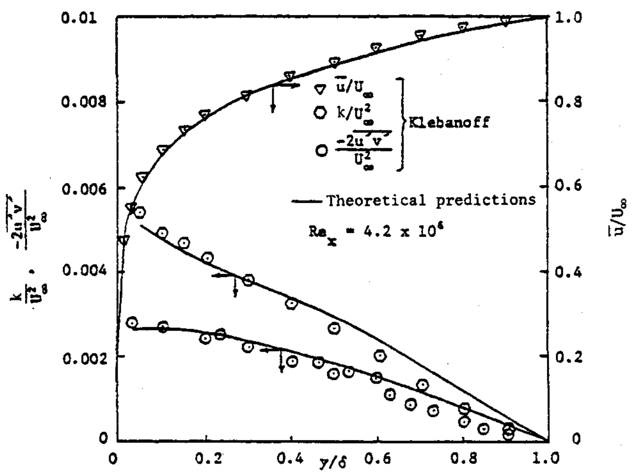

Fig. 1 Comparison of calculated turbulent boundary-layer results with measurements of Klebanoff.[6]

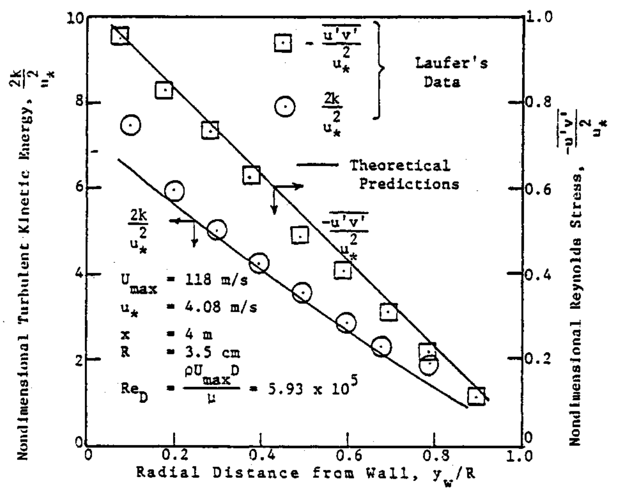

Fig. 3 Comparison of calculated turbulent boundary-layer results with measurements of Laufer.[10]

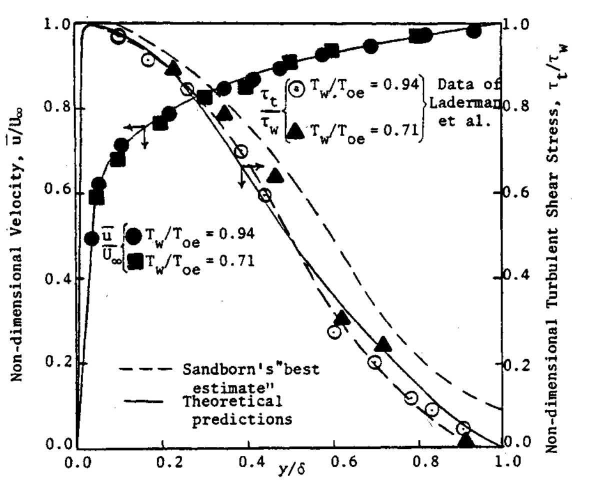

Fig. 2 Comparison of calculated compressible turbulent boundary-layer results with measurements of Laderman et al.[7,8] and with Sandborn's "best estimate."[9]

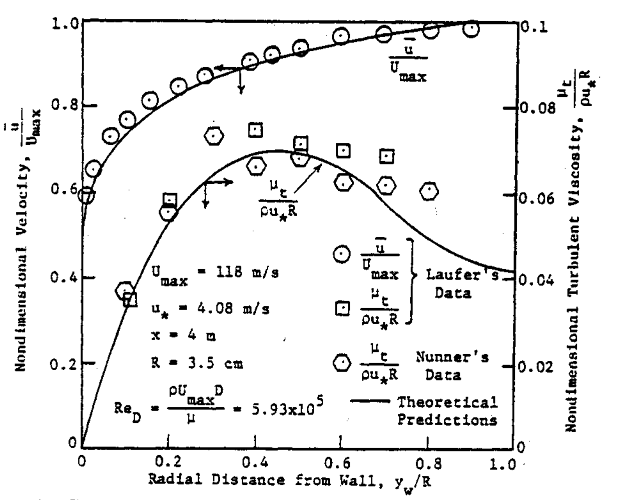

Fig. 4 Comparison of calculated turbulent boundary-layer results with measurements of Laufer[10] and Nunner.[11]

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